OCR
RECOGNISING ACHIEVEMENT

ADVANCED SUBSIDIARY GCE **4725/01**

MATHEMATICS

Further Pure Mathematics 1

FRIDAY 11 JANUARY 2008 Morning

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
List of Formulae (MF1)

INSTRUCTIONS TO CANDIDATES

- Write your name, centre number and candidate number in the spaces provided on the answer booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- **You are reminded of the need for clear presentation in your answers.**

This document consists of **4** printed pages.

1 The transformation S is a shear with the y-axis invariant (i.e. a shear parallel to the y-axis). It is given that the image of the point $(1, 1)$ is the point $(1, 0)$.

(i) Draw a diagram showing the image of the unit square under the transformation S. [2]

(ii) Write down the matrix that represents S. [2]

2 Given that $\sum_{r=1}^{n}(ar^2 + b) \equiv n(2n^2 + 3n - 2)$, find the values of the constants a and b. [5]

3 The cubic equation $2x^3 - 3x^2 + 24x + 7 = 0$ has roots α, β and γ.

(i) Use the substitution $x = \frac{1}{u}$ to find a cubic equation in u with integer coefficients. [2]

(ii) Hence, or otherwise, find the value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$. [2]

4 The complex number $3 - 4\mathrm{i}$ is denoted by z. Giving your answers in the form $x + \mathrm{i}y$, and showing clearly how you obtain them, find

(i) $2z + 5z^*$, [2]

(ii) $(z - \mathrm{i})^2$, [3]

(iii) $\frac{3}{z}$. [3]

5 The matrices **A**, **B** and **C** are given by $\mathbf{A} = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 4 \\ 0 \\ 3 \end{pmatrix}$ and $\mathbf{C} = (2 \quad 4 \quad -1)$. Find

(i) $\mathbf{A} - 4\mathbf{B}$, [2]

(ii) $\mathbf{BC}$, [4]

(iii) $\mathbf{CA}$. [2]

6 The loci C_1 and C_2 are given by

$$|z| = |z - 4\mathrm{i}| \qquad \text{and} \qquad \arg z = \tfrac{1}{6}\pi$$

respectively.

(i) Sketch, on a single Argand diagram, the loci C_1 and C_2. [5]

(ii) Hence find, in the form $x + \mathrm{i}y$, the complex number represented by the point of intersection of C_1 and C_2. [3]

7 The matrix $\mathbf{A}$ is given by $\mathbf{A} = \begin{pmatrix} a & 3 \\ -2 & 1 \end{pmatrix}$.

(i) Given that $\mathbf{A}$ is singular, find a. [2]

(ii) Given instead that $\mathbf{A}$ is non-singular, find $\mathbf{A}^{-1}$ and hence solve the simultaneous equations

$$\begin{aligned} ax + 3y &= 1, \\ -2x + y &= -1. \end{aligned}$$

[5]

8 The sequence $u_1, u_2, u_3, \ldots$ is defined by $u_1 = 1$ and $u_{n+1} = u_n + 2n + 1$.

(i) Show that $u_4 = 16$. [2]

(ii) Hence suggest an expression for u_n. [1]

(iii) Use induction to prove that your answer to part **(ii)** is correct. [4]

9 **(i)** Show that $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta)$. [2]

(ii) The quadratic equation $x^2 - 5x + 7 = 0$ has roots α and β. Find a quadratic equation with roots α^3 and β^3. [6]

10 **(i)** Show that $\dfrac{2}{r} - \dfrac{1}{r+1} - \dfrac{1}{r+2} = \dfrac{3r+4}{r(r+1)(r+2)}$. [2]

(ii) Hence find an expression, in terms of n, for

$$\sum_{r=1}^{n} \frac{3r+4}{r(r+1)(r+2)}.$$

[6]

(iii) Hence write down the value of $\displaystyle\sum_{r=1}^{\infty} \frac{3r+4}{r(r+1)(r+2)}$. [1]

(iv) Given that $\displaystyle\sum_{r=N+1}^{\infty} \frac{3r+4}{r(r+1)(r+2)} = \frac{7}{10}$, find the value of N. [4]